Solving the Problem of Logical Omniscience

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Abstract: I examine three ways of addressing probabilism's implausible requirement of logical omniscience. The first, and most common strategy is to say that it's plausible to require an *ideally rational* person to be logically omniscient. I argue there is no sense of "ideally rational" on which this view is defensible. The second strategy says probabilism should be formulated in terms of not logically but doxastically possible worlds, ways you think the world might be. I argue that, on the interpretation of this approach that lifts the requirement that we believe all remote logical truths, the view becomes vacuous, issuing no requirements on rational believers at all. Finally, I endorse and develop a new solution to the problem. This view proposes dynamic norms for reasoning with credences. The solution is based on an old proposal of Ian Hacking's that says you're only required to be sensitive to logical facts when you know they are logical facts.

I. The Problem

Different things get called "the problem of logical omniscience". Here I'll be concerned with a problem for the view known as probabilism.

Probabilism is a necessary condition on rational credence. It says that, necessarily, a person's credences are rational only if they obey the axioms (and consequent laws) of probability theory.

We'll formulate the axioms in the following standard way:

> (A1 Logicality): the probability of any logical truth is 1.
> (A2 Non-negativity): every proposition has a non-negative probability.
> (A3 Additivity): the probability of a disjunction with logically exclusive disjuncts is the sum of the probabilities of the disjuncts.

Some notable laws that follow from this include:

> (L-Entailment): if q is a logical consequence of p, the probability of q is no lower than the probability of p.
> (L-Partition): propositions in a logical partition receive probabilities summing to 1.

Probabilism thus requires any rational person to be extremely sensitive to logical truth and consequence. Just by the first axiom, the rational credence in any logical truth is *always* 1, the maximal credence, normally interpreted as *certainty*. This is the problem of logical omniscience. It doesn't seem that any rational person has to always be certain of every logical truth, or as confident of a consequence as she is of any proposition that entails it. Was it irrational to be less than certain of Fermat's Theorem in the past, or even to be less confident than we are of the Peano Axioms? Is it irrational even now? (The first "proof" Andrew Wiles publicly presented had an error in it.) Other manifestations of the problem are the requirement, by the L-Entailment law, to give logically equivalent propositions exactly the same credence. Was it irrational to give the Axiom of Choice and the Well-Ordering Theorem and Zorn's lemma distinct credences before their logical equivalence was proved? Intuitively, it is *possible* for *someone* at *some time* to have *rationally* given *some* pair of equivalent propositions distinct credences.

While my intuitions tell strongly against the view that rationality requires such "logical omniscience", some other people's intuitions favor the requirement. Some simply intuit that you must accept any logical consequence of what you know. A more modest intuition many more people have is that you at least must accept a consequence that follows by a "simple" deductive step, such as a step by Modus Ponens. (See, e.g., Steinberger pp.10-11 and his references there to Broome and MacFarlane.) Though that latter intuition looks more modest, it still entails that anyone who fails to accept all logical consequences of what they know is committing some failure of rationality. This is because, as we know, we can fashion a deduction system that prescribes only simple steps and that is "complete", i.e. every logical consequence can be reached by a sequence of such steps. So, if it's intuitive that any "simple step" is required, then the perfectly rational person (one who is guilty of no failure of rationality) must be logically omniscient, and so probabilism's requirement on credences ends up supported by intuition, even the more modest intuition.

There are not only intuitions but also arguments in favor of logical omniscience as a rational requirement. In particular, there is an argument in favor of probabilism that's very popular right now, the so-called "accuracy" argument. (Joyce, Pettigrew) This argument aims to show that anyone who violates probabilism has less accurate attitudes than does someone who obeys probabilism.

And alongside the intuitions and arguments in favor of logical omniscience or probabilism, there are also defenses of the logical omniscience requirement. Christensen and others defend (as well as argue for) the view that logical omniscience is a fine requirement so long as we understand that it concerns the *ideally rational* person.

Some people who appreciate the problem with probabilism's requirement of logical omniscience have looked for better theories that save what's good in probabilism while removing the logical omniscience requirement.

In a recent paper, Robbie Williams motivates and develops such a view, a view he calls doxastic probabilism. The idea of this view, which I'll explain more below, is to replace logical possibilities with a person's "doxastic" possibilities in the formulation of probabilism.

And much earlier Ian Hacking offered a suggestion for fixing up the logical omniscience requirement from probabilism. He suggested that you should obey axiom (A1) only when you *know* that the proposition is a logical truth, and you should obey (A3) when you *know* that the two disjuncts are mutually exclusive.

My plan for this paper:

In section II, I'll say why the accuracy argument doesn't support probabilism.

In section III, I'll criticize interpretations of probabilism as a condition only on *ideal* rationality, and in doing so I'll critically discuss the ("modest") intuition I mentioned some people have that seems to support probabilism.

In section IV, I'll present Williams's doxastic probabilism and its motivations, and I'll explain why I don't favor taking that approach.

In section V, I'll sketch a new solution to the problem that builds on Hacking's view. I'll incorporate Hacking's idea, together with another important idea I'll take from Gilbert Harman, into a larger framework that says credences in logical truths must obey "dynamic" norms of rationality, that is, norms that concern how we revise our credences by reasoning over time. So, for example, while I reject probabilism's requirement that we *always* be certain of every logical truth, I will endorse, among other things, some norms that tell us to revise our credences as we acquire proofs that give us knowledge that particular propositions are logical truths.

II. The Accuracy Argument

The argument that any violator of probabilism is less accurate than some follower of probabilism has valuable contributions to offer epistemology. But, on its own, it just doesn't support probabilism. It especially clearly doesn't support probabilism's logical omniscience requirement on rationality.

The disconnect between the argument's conclusion and probabilism is due to a simple and familiar point: a rational attitude is a very different thing from an accurate attitude or even a provably accurate attitude. The point is most familiar and clear when it's made about full belief in traditional epistemology. It's just implausible that, necessarily, a rational person must believe, much less know, every logical truth. If you are unaware of any proof of Fermat's theorem, believing it is *not* rational. (And, as the next section will discuss, it's not a requirement *of rationality* that you *produce the proof*.) For anyone who feels it's objectionable for a theory of rationality to require full belief in all logical truths, it will be no reassurance at all to hear that being logically omniscient would give you provably accurate beliefs. That was not unclear. Likewise, for anyone who feels it's a problem for a theory of rationality to require full *credence* in every logical truth, it's simply no reassurance to hear the accuracy argument for probabilism.

That's my dismissal of the accuracy argument as a positive argument for probabilism and especially for logical omniscience. It's brief, I know. But what can be said against it? If the accuracy argument and probabilism, with its requirement of logical omniscience, are to be defended against my dismissal, the defenders will, I believe, turn to some appeal to the notion of ideal rationality. So let's examine that now.

III. Probabilism and Ideal Rationality

Some people say that *ideal rationality* requires a person to have both full beliefs and maximum credences in all logical truths. The idea is widely popular (Talbott says in his Stanford entry on Bayesian Epistemology that "most Bayesians maintain the assumption of logical omniscience and treat it as an ideal to which human beings can only more or less approximate", sec. 6.1.A), but I will focus on the two most extensive recent defenses by David Christensen and Declan Smithies.

What is "ideal rationality"? There are two readings we can distinguish.

On a first reading, ideal rationality is the subject matter of a theory that *idealizes* in the sense of abstracting away from, or ignoring, certain real world constraints. It's the sense of "idealize" in which we sometimes ignore friction in a physics calculation. What might the epistemologist similarly ignore? Candidates include the limits of human memory, the limited amount of time we have to think and to come up with proofs and arguments or (relatedly) the limited speed at which we think or perform calculations, or the limited stock of concepts we possess or (relatedly) the limited range of hypotheses we can take under consideration at a given time.

In defense of idealizing in this way, I see only one possible point: it has the advantage of making our work easier. But that is only a pragmatic defense. It does nothing to show, for any problematic feature that might crop up in the theory that results from such easy work, that the problem is not really a problem; the problem remains just as much a problem. This pragmatic excuse has to be followed up with some reassurance that in the *true* theory, this problematic feature will disappear. So the first reading doesn't really get anywhere with the problem of logical omniscience. (Smithies also distinguishes and sets aside this first reading, 2774.)

The second, more promising, reading of the notion of ideal rationality is the notion of the *ideally*, or *perfectly*, rational person. It is the notion of a person who obeys every instance of every norm of rationality, without a single failure or falling short in any way. If any norm calls on us to do something to the "maximum" extent, the ideally rational person does that thing to that maximum extent. Or if a norm says that the more you do something the better, then the notion of the ideally rational person here is like a limit case of doing more and more of that thing, even if there is no "maximum".

Is there any reason to think the ideally rational thinker, ideal in our *second* sense, is ideal in that *first* sense, that is, has no limits on things like memory, speed, computational power, or conceptual repertoire? No, I see no reason to believe that the thinker who follows the norms of rationality to the fullest, without any error, will have unlimited memory, speed, computing power, concepts, or any such thing. This claim (that the second sense of 'ideal' entails the first sense) would require some argument that there is a norm *of rationality* that prescribes having a better memory or prescribes reasoning faster or crunching through more proofs or calculations. But any norm that says something like "Calculate *faster!*" is not a norm *of rationality*. It leads to a theory of the ideally *fast* thinker or calculator, not the ideally *rational* thinker. (I'm not saying that some norm or ideal couldn't require your being as fast as possible at one thing or another. Brian Knab points out to me that it might be true that the ideal *runner* is "maximally" fast. I'm just claiming that nothing like that is true *of rationality*.)

I have no *argument* in defense of my claim about what rationality is (not) about. I don't rest that claim on firmer premises. I'm just appealing to your understanding of our concept of rationality. But I think, once pointed out, it's an intuitive claim. Thus, we must avoid, on pain of changing the subject, assuming that the ideally rational thinker, in our present second sense, has unlimited memory, time or speed, computational power, or any such thing.

In fact, I can't see how there is *any* norm that tells us we ought, in *any* sense of "ought", to do every calculation or computation possible. It's true that an ideally fast and thorough calculator might produce perform every calculation super fast, but that doesn't mean that anyone or anything *ought to* do that. (Consider the ideally fast singer or dancer, someone who dances or sings "maximally" fast. That's not something anyone *ought to do!*) So, I think that unwisely assuming the ideally rational thinker in the second sense is ideal in the first sense would actually change the subject not just away from *rationality*, but away from normativity entirely.

Which of these two notions of ideality do Bayesians actually have in mind when they defend probabilism, and logical omniscience in particular, as a norm of "ideally rational belief"? Smithies explicitly opts for the second (2774). Christensen doesn't explicitly draw the distinction we've just drawn, but I'm confident the correct view to assign him is this second interpretation. He often talks of "the standard of absolute rational perfection" (153), and this second reading often makes the best sense of his defense of probabilism.

So, now, why think that an ideally rational thinker (in the second sense) will be logically omniscient? Some people (among them Christensen cites Ian Hacking, Richard Foley, and Philip Kitcher) worry that requiring logical omniscience is as implausible as saying the ideally rational person is omniscient period, including "factually" omniscient (i.e. knows all non-logical truths). Christensen responds to that worry with an example designed to show that rationality does require us to believe logical consequences (of what we rationally accept) in a way it does not require us to believe all non-logical truths. In the example, one person, Cherry is rationally confident that (P1) anyone near a Grizzly cub in the wild is in danger, while another person, Kelly, is likewise rationally confident of P1 but also of something Cherry has no confidence in, (P2) Kelly is herself near a Grizzly cub in the wild. Cherry and Kelly both fail to be confident of the logical consequence of P1 and P2 that Kelly is in danger. But, Christensen points out, obviously this is a serious rational failure on Kelly's part and no rational failure at all on Cherry's part. (154-5.)

Christensen doesn't just use the example to *defend* against the worry (that requiring logical omniscience is as bad as requiring omniscience period). He suggests it *positively supports* the claim that logical omniscience is a rational requirement. After presenting the case of Kelly's mismanaged response to her evidence, he says that "logical omniscience emerges naturally as the limiting case of one of the basic ingredients of good thinking. … Eliminating this sort of mistake [failing to respect logical relationships, as Kelly did] yields, in the limit, logical omniscience." (156) Smithies endorses Christensen's use of the Kelly example and draws the same conclusion that Christensen does (sec. 2). The thought Christensen and Smithies have here seems be an elaboration of the "modest" intuition that I brought up at the start of this paper, an intuition some people have and that can be leveraged to support logical omniscience as a rational requirement: according to this intuition that many people have, we're required to make any simple logical inference, and since every consequence follows by some sequence of simple steps, we are required to be logically omniscient.

I don't think this is right. I think Christensen and Smithies generalize from Kelly's case to the requirement for full logical omniscience too quickly. There are epistemically significant differences between the particular logical consequence we intuitively feel Kelly must accept and other logical consequences like Fermat's theorem.

The important difference, I suggest, is that Kelly is, or ought to be, *interested* in whether she's in danger, while (we can suppose) she currently doesn't have, and it's not the case she now ought to have, any interest in Fermat's theorem. So, the proposal is that your interests (or attention, if you prefer)—what your interests are as well as what they ought to be—make an epistemically significant difference. (I won't argue over the correct interpretation of "ought" here, since a pragmatic or prudential reading offers us at least one attractive option and we don't have to pick now.)

The proposal I want to endorse is due to Gilbert Harman. He famously observed that there is a negative norm on good thinking: *avoid clutter!* (12) Don't infer logical consequences when it would only clutter your mind (e.g. inferring disjunctions like that birds fly or the moon is cheese). The clutter avoidance norm explains why, as we said, ideality in the present second sense doesn't entail ideality in our earlier first sense. Don't waste time producing proofs or calculations of clutter either! But, something relevant to the Kelly example is a very important caveat Harman immediately added after he introduced the clutter avoidance norm, a caveat that's often missed or forgotten: if you are, or ought to be, interested in something you recognize is a consequence (of what you already believe), then you are *obliged* to believe it. (15, see also 55; Harman credited this caveat to Stalnaker.) With this caveat included, Harman's full proposal offers a more plausible view than the view that an ideally rational person, one who obeys every instance of every rational norm, is logically omniscient. Kelly ought to be interested in the proposition that she's in danger, and her acceptance of P1 and P2 means she recognizes that that proposition follows from things she knows. Since Harman's view fully explains the Kelly example, the example is no help to the case for, or in defense of, logical omniscience.[1]

The Harman proposal is attractive also because it makes sense of that intuition that logical omniscience and "factual" omniscience ought to be treated analogously in the theory of rationality. The Harman proposal, on my favored understanding, says that there are things you are in a position to and *merely permitted* to know or rationally believe, but there is no obligation, and there is some reason (namely, clutter avoidance) to not form the belief. But this is analogous for beliefs in logical consequences and for beliefs you are in a position to perceptually or introspectively justifiably adopt. I'm in a position to form many beliefs about my environment or my mind by meditating on the evidence constituted by my current experience, but I don't have to form those beliefs to be rational. But if I do, or if I ought to, take an interest in (or attend) to certain experiences, maybe while walking along a busy sidewalk, then I'm in a situation like Kelly's—then it is irrational to not believe these things I was previously merely permitted to (was in a position to justifiably) believe.

To make my position clear, let me put down an official statement of the proposal form Harman as I interpret it and intend to endorse it.

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[1] Jane Friedman, forthcoming, gives a valuable recent examination of Harman's clutter avoidance principle. She raises important critical points about the clutter avoidance principle and its surprisingly significant implications that I can't take up here.

(Harman's Interest Proposal): If you have no interest, and ought have no interest, in *p*, then you're not rationally required to believe *p* (though you may be rationally permitted). If you do or ought to have an interest in *p*, and *p* is a *simple* consequence or a *recognized* consequence of your (undefeated) knowledge, then you are required to believe *p.*

I'll leave "simple consequence" undefined. The intuitive example is something the follows by a single application of Modus Ponens; a full definition might enumerate such familiar rules of derivation.[2]

We can now also criticize another example Christensen brings up to support his claim that ideal rationality "in the limit" requires logical omniscience. He suggests that ideal rationality is analogous to ideal chess strategy. And he argues that we should concede that ideal chess play is beyond any living human's abilities. We would of course hesitate to assert that Garry Kasparov is not a good chess player, but we can admit that he is not an *ideally* good chess player. Christensen rightly points out that we easily understand the sense in which he, or any real person, falls short of the ideal. These claims about ideal chess play sounds reasonable, but I claim they are not a good analogy to ideal rationality, especially once we consider things in light of Harman's proposal. A chess player is *forced* to make some move on each turn, but a believer is not similarly forced, so there is a room to say an ideally rational believer just doesn't "make any move", doesn't form any belief, in many cases, especially when it would result in clutter. So, the chess analogy doesn't support the case for logical omniscience.

Smithies adds a further argument for logical omniscience as a norm on ideal rationality. Smithies's case for requiring logical omniscience can be responded to in a way similar to how I've responded to Christensen's already.

In order to make his case for logical omniscience as a requirement of rationality, Smithies devotes most of his effort to supporting the idea that we all have justification, specifically what he calls *propositional justification*, for all logical truths. (His defense involves arguing that the logical facts themselves, not any experience or mental state of ours, constitute our justification for the logical truths, and therefore constitute justification for logical omniscience.) From there, he then goes on to make the claim that ideal agents hold *doxastically justified* beliefs in all those logical truths that we enjoy propositional justification for. (Ordinary humans, though, are of course incapable of believing all the infinitely many things they have propositional justification to believe, and Smithies also argues that, even for many of the logical truths we can believe, we non-ideal humans are incapable of enjoying doxastic justification for them.)

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[2] In helpful comments on an earlier draft, Declan Smithies pointed out to me that, while Harman's proposal does allow me to handle the Kelly example while denying that (ideal) rationality requires logical omniscience, a fan of the logical omniscience requirement like him might also want to accept something very close to Harman's proposal. Smithies mentioned that, for example, one could say that, for *any* logical truth or consequence, until you are interested in it, you're not required to believe it, but once you do or ought to take an interest in it, you're required to believe it, even if it is a proposition you don't recognize is a consequence of things you know, even if it is a very remote consequence like Fermat's theorem. (That of course differs a bit from the indented formulation I endorsed.) Harman's broader proposal to tie rational requirements to interests isn't my motivation for rejecting logical omniscience as a rational requirement; it's my means of addressing my independent motivation.

My response to Smithies is to question his move from "we have propositional justification to believe …" to "an ideally rational person is required to believe …". This is a critical step he takes, but does not argue for.[3] The step seems to me to be a mistake, and avoiding it avoids the problem of logical omniscience. Again, we should say we have mere permission, without obligation, to believe things. This should be the situation with the logical truths Smithies says we have propositional justification to believe.

Do we have propositional justification for all logical truths? I think it's a terminological question, since "propositional justification" is a term of art. We might define "propositional justification" in ordinary terms as "what we have sufficient support for rationally believing, whether or not we do rationally believe it". That doesn't leave me 100% comfortable saying we have propositional justification for logical omniscience, but I'm not closed to the view. Schoenfield 2012 usefully distinguishes (a) what we have support for (given the evidence), and (b) what we rationally ought to believe, i.e. are required to believe. Again, I'm not 100% comfortable saying (a) applies to all logical truths and consequences, while (b) doesn't—but I'll leave the position open.

In any case, it doesn't matter. The strategy of saying that we have propositional justification for logical omniscience does not save probabilism, which says an (ideally) rational person is *required* to be logically omniscient. This is explicit in the formulations by Christensen (viii, 106-7) Joyce, Pettigrew, Hajek, Titelbaum textbook, Bradley textbook, Meacham 2014 p. 1186, Weisberg, Briggs 2009 p.60, and every formulation of probabilism I've ever seen. I have never seen a Bayesian endorse probabilism or logical omniscience by putting it in terms of a *permission*. However we want to define technical terms like "propositional" and "doxastic justification", the thing that's important to me is just this: *if* we have permission to be logically omniscient, it is a *mere* permission, and *that* is what makes probabilism false. Wherever you want to fit "propositional justification" into this position is fine with me.

I'll incorporate Harman's proposal (that we have mere permissions that our interests can convert into requirements) as an important part of the positive proposal I'll endorse in the final section of this paper. But first, the next section looks at a different approach to the problem of logical omniscience.

IV. Replacing Logical Possibility with Doxastic Possibility

In a valuable recent paper, Robbie Williams motivates and presents a new view to replace traditional probabilism. His motivation is an instance of the more general problem of logical omniscience. He's concerned with instances of probabilism's implausible demand that our credences in all logical truths always be 1. But the instances that Williams focuses on are

[3] Smithies makes the undefended step when he says: "My working assumption is that facts about rationality are explained by facts about reasons or justification. More specifically, I assume that rationality **requires** one to believe a proposition just when and because one has sufficient reason or justification to believe that proposition." 2775, bold added. (On page 2782, Smithies makes clear that the above language concerns propositional justification, "Propositional justification is a matter of having reasons or justification to believe a proposition... .") Smithies also again explicitly says that an ideally rational agent has a doxastically justified belief in every proposition she has propositional justification for on p.2789.

interestingly different from the ones I started this paper with. He does not have in mind remote, hard-to-prove logical truths or consequences like Fermat's theorem or Zorn's lemma. Rather, to motivate his project, he argues against probabilism's requirement that we must be certain even of *elementary* logical truths, like instances of P v ~P. With this as his motivation, he proposes a view he calls *doxastic probabilism*. I want to examine doxastic probabilism because the view very naturally looks like it promises to overcome the whole problem of logical omniscience, including our worries about the demand to be certain of "remote" logical truths like Fermat's theorem.[4] I'll present Williams's motivation, present his view, and raise a worry about the view.

Williams's motivation is this. Logical paradoxes, like the Liar paradox, have generated different reasonable, or at least non-crazy, responses from philosophers. One such response declares that propositions of the form P v ~P are not all logical truths and that some paradox-generating instances of it should receive a credence of 0 (e.g. this is like Hartry Field's view). Other responses leave the status of all instances of P v ~P in tact as logical truths and give them a credence of 1. Williams says that it could be perfectly reasonable to be agnostic about which of these philosophical approaches is on the right track. Just as a probabilistic framework can recommend a middling credence in the face of uncertainty over some conventionally contingent matter like a proposition stating the location of, say, a stolen car (has it left the state?), so likewise, Williams says, a sensible probabilistic framework should be able to also recommend a middling credence in the face of uncertainty over where the logical truth lies. Since this isn't allowed by traditional probabilism, which Williams calls *logical probabilism*, we need a new view.

The new view that Williams proposes, doxastic probabilism, is achieved by replacing the role of *logical possibilities* in the standard view with something he calls *doxastic possibilities*. (See his sec. 7.) To understand the resulting view, it's thus obviously central to understand what this new technical notion of a doxastic possibility is. But Williams actually doesn't define the notion. He says he will "[t]ake as a working primitive the notion of an agent's *doxastic space*, the set of worlds that are *doxastically possible* for her." However, he does put a constraint on the notion. He says he will "[a]ssume the following: <u>For [person] *z*, it must be that *p* iff *p* is true at each [world] *w* doxastically possible for *z*</u>." (p.6, underlining added for readability here.) So, Williams gives us a biconditional linking doxastic possibilities with certain epistemic modals, "must" claims. I take it we can also restate the constraint using "might" claims. That is, I take it the following biconditional holds too: <u>for a person, it might be that *p* iff *p* is true at some world that is a doxastic possibility for her</u>. I also take it we could re-organize things to isolate the notion of doxastic possibility on one side of the biconditional: <u>a world *w* is a doxastic possibility for a person iff there is a proposition *p* that might be true for her and *w* is true at *p*</u>.

It will be important to my worry that we correctly understand the laxity of the conditions for correctly attributing the attitudes we express using these epistemic modals. For some people,

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[4] After kindly reading a draft, Williams said that he's wary of claiming his view will solve the *whole* problem of logical omniscience, where the "whole" problem would include both his own motivation about rational low credence in "elementary" truths and our motivation about rational low credence in "remote" truths. (See footnote 3 below for a bit more on why Williams is wary of claiming his view solves the whole problem.) I should repeat, to be clear, that in his paper Williams didn't take our problem of rational low credence in "remote" truths as his motivation. I just say it would be natural to think his view promises to solve the whole logical omniscience problem, and Williams also agrees that many readers may naturally think so, even though he is wary of this himself.

at some times, what might be true includes logical impossibilities, and any inconsistent combination of logical possibilities (e.g., P together with ~P; or P, ~Q, and if-P-then-Q all together). There are no apparent prohibitions on what might be true, and certainly at least no logical restrictions.

We can now ask exactly what requirements doxastic probabilism puts on a rational person's credences. What we'd like to know is how to convert the traditional axioms of probability into norms stated using doxastic worlds, since doxastic probabilism will then just be the requirement that a rational person's credences obey those norms (and all their consequent requirements).

But what exactly do the re-written axioms look like?

There's no need to revise the non-negativity axiom, but the others will need to be revised.

The original logicality axiom was this (with some emphasis added):

A1 (Logicality): the probability of *any logical truth* is 1.

What is the new rule for doxastic probabilism? It will be this:

D1 (Doxastic logicality): the rational credence in *anything that must be true for you* is 1.

Here is my worry about this. I don't see how anyone could clearly violate this rule. Is anyone rationally required to be certain of anything on this view? Imagine some possible examples. Suppose I've invited my student to consider Descartes's proof: you are a thinking and doubting thing, and therefore you exist. And suppose my student is certain of the premise, but remains uncertain of the conclusion. This student is off the hook, so long as *for him*, it *might be* that he doesn't exist, even while he remains certain he is thinking and doubting (that he exists!). Or suppose we're in logic class, and we've proved a completeness theorem for propositional logic, but a student is withholding her certainty. On doxastic probabilism, there is nothing irrational about being uncertain even after fully understanding a correct proof and being certain of all its premises, because the student can always say, and can rightly say, for her, it *might be* that the conclusion is false. We all always have the potential to *rightly* say this because doxastic worlds are constrained only by epistemic modals, and it is up to each person what might be true for them. By "up to" I don't mean we choose (maybe we can, I don't know); I mean there are *no* rational requirements, no prohibitions, against this or that proposition being something that might be true for you. Certainly, Williams suggests no such rational requirements, and I don't see how there could be any if our ultimate *motivation* is to permit violations of the "correct" logic. Williams's own original motivation was an instance of excluded middle, and Williams thinks such an instance *might* be false even if classical logic is the "correct" logic. So, any proposition, as far as I can see, could be such that it might be false for you or me or whoever, and we have to say there is nothing irrational about it.[5]

Next, we want to know how the additivity axiom goes under doxastic probabilism. The simplest suggestion would seem to be this:

> D3* (a first pass): if, for you, no doxastic world makes true both disjuncts of a disjunction, the rational credence in the disjunction is the sum of your rational credences in its disjuncts.

But, as Williams observes in another paper (Williams 2016), for some familiar ways of rejecting classical logic, or rejecting classical semantics, D3* is not an attractive principle. There is the supervaluationist who thinks the disjunction, "Patchy is red or not red", is true while each disjunct is not true, because Patchy is a borderline shade of red. Such a supervaluationist will want to reject D3* as well as anything too closely resembling traditional additivity. (They will instead want a sort of principle often called "subadditivity".) So, D3* doesn't help us see what principles a doxastic probabilist should propose.

A better option for the doxastic probabilist might be to reformulate D3* so that it isn't explicitly about disjunctions. An idea for a way to do that can be drawn from Williams 2016. Instead of talking of disjunctions as such, we can talk about propositions that are partitioned by other propositions, where the partitioning is not classical logical partitioning. Here is a proposal (I'm drawing this from a more general technical discussion in Williams 2016, sec. 3) for a rule for how doxastic probabilists might, in effect, handle disjunctions. Think of *r* as serving as the disjunction of *p* and *q* in the following rule.

> D3 (Doxastic additivity): <u>if</u>, supposing *r* is true then one of *p* or *q* must be true for you, and, (still supposing *r* is true) it's not the case that both of *p* and *q* might be true for you, <u>then</u>, if you're rational, your credence in *r* is the sum of your credences in *p* and in *q*.

My worry, though, is that, as with D1 earlier, D3 cannot be violated. To see this, let's consider when it is supposed to be violable.

We'll get cases where D3 and A3 issue differing permissions in cases where two "disjuncts", *p* and *q*, are both true is no logically possible world but are true in a doxastically possible world (for you). While A3 would then require you to give the disjunction the sum of the disjuncts' credences, D3 will permit you to give a lower credence. (This is one way of letting P v ~P have a credence less than 1.) My worry, though, is that D3 is permissive here because it is *always* permissive. Any case where it would initially *appear* someone is violating it will be

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[5] A note for careful readers of Williams's paper: Williams, on p.9, while discussing "recognition requirements" of other authors, says that if a student *recognizes* that some theorem follows from premises she's certain of, then she is required to be certain of the theorem, because recognizing entails doxastic certainty. However, nothing requires our student to *recognize* anything. In particular, doxastic probabilism issues no such requirement. Again, there are no requirements, at least no plausible such requirements are forthcoming or have been suggested, to constrain the rationality of attitudes expressed by epistemic modals.

ultimately better described, and *correctly* described, as a case where they are not violating it. Suppose I initially appear to violate D3 by saying I think there's half a chance of rain tomorrow and half a chance of snow and I think there's three quarters of a chance of either rain or snow. Any such case will be better described as one where I turn out to think it might both rain and snow, and thus there are simply more doxastic possibilities (dreamt of in my philosophy) than it first appeared when it appeared I was violating D3. So, I worry D3 will *never* require anyone to give a disjunction a credence as high as the sum of its disjuncts.

The source of the general problem here is that the only grounds we've been given for attributions of doxastic possibilities are attributions of epistemic modals, and any apparent violation of this axiom (by giving the disjunction less credence than the sum of the disjuncts) will itself be sufficient reason for attribution of an epistemic modal, a might claim that the disjuncts are both true, which will thus also amount to an attribution of doxastic possibilities where both disjuncts are true. The source of the problem is a feature of doxastic probabilism that Williams is explicit about when he says: "the true rational constraints, I contend, govern the agent's partial beliefs together with their modal attitudes—what is possible, and what is impossible, by their lights." (p.7) What makes this problematic, to my mind, is that between these two moving pieces that the rational constraints always jointly govern, it will always be an option, and always make most sense, and thus always be *correct*, to attribute attitudes expressed by modals such that the partial beliefs don't violate the probabilistic constraints. I don't have an argument for this claim that my objection rests on, but I think it is highly plausible: it is never correct to attribute irrationality when the alternative option carries no cost at all but the attribution of a doxastic possibility.

Reasons to deny a view are, like all reasons, defeasible, so my objection comes with a constructive suggestion for those who disagree with me. My constructive suggestion would be: if you wanted to stick with doxastic probabilism in the face of my worry, I suggest the way to defend it would have to be to develop some plausible constraints on the rationality of what doxastic possibilities a person has, where these constraints are wholly independent of probabilism and its motivations. My view is that this isn't the most promising approach to salvaging probabilism, but of course this depends on a comparison with the alternatives, including the one I want to advocate below.[6]

I talked about whether you could violate D3 by giving a disjunction too *low* a credence, but, to indicate the generality of my worry, let me also quickly mention the alternative. Does doxastic probabilism issue a requirement that we not give a disjunction a credence *higher* than the sum of its disjuncts? No, D3 doesn't prohibit this either. Williams wants doxastic probabilism to let us give credence to any solution to the logical paradoxes. It is designed to accommodate even someone like the supervaluationist already mentioned earlier, someone who suspects it might be that P v ~P is true even though each disjunct receives zero credence: for the disjunction could simply be one of my doxastic possibilities while neither disjunct is. In general, doxastic probabilism is motivated by the idea that there is no restriction on what violations of even elementary logic it's reasonable to entertain.[7]

V. Developing a Dynamics for Rational Credence[8]

Williams proposes that probabilistic norms be anchored to the attitudes we express and attribute using the epistemic modals "might" and "must", and my worry has been that this anchor doesn't catch any firm ground. A similar but crucially different approach anchors probabilistic norms to *knowledge* instead of to those "might" and "must" attitudes. (Williams is explicit that this "knowledge" view is not his view. On p.9, last two paragraphs, he says what "must" be for a person is not the same as what they know.) Such a knowledge-anchored view was first proposed by Ian Hacking in an important paper on the problem of logical omniscience from 1967. My experience is that some formal epistemologists will say in conversation that they like Hacking's proposal, but no one to my knowledge has tried to develop it or has even endorsed the view in print. I'll describe Hacking's proposal, I'll offer some speculation as to why Bayesians have been

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[6] In his helpful comments on a draft, Williams agreed that it's a challenge for him to say what further facts constrain a persons' doxastic possibilities in such a way that her credences may violate doxastic probabilism. But he's optimistic about meeting the challenge, and in his comments he suggested some possible resources for meeting it. For a first example, Williams suggested that one factor that could help constrain what doxastic possibilities a person *has* are the facts about what doxastic possibilities are *justified* for her by the genuine *reasons* she has. (I understand the suggestion to be reminiscent of one component of David Lewis's views on what determines what attitudes a person has.) For a second example, Williams suggested that a person may *accept* (in a semi-technical sense) a particular logic, where what logic a person currently accepts is not settled by her current beliefs or credences, and her acceptance of a logic means that her doxastic possibilities contain only the possibilities of that logic. Thus, someone who accepts classical logic won't have any doxastic possibilities in which P & ~P is true, and this may be so even if, at the moment, she is sincerely asserting that P & ~P might be true. Her assertion will then be mistaken, Williams suggested.

As mentioned in an earlier footnote, Williams still doesn't take this to help solve the whole problem of logical omniscience. Someone who "accepts" classical logic and is confident of the Peano Axioms will now face a counter-intuitive requirement to be confident of Fermat's theorem. As Williams fairly describes the predicament: either he can decline to meet the challenge, and his view will lack any bite in the way that I worry in the main text, or else he can take up the challenge as he wishes to, but that then prevents the resulting view from solving our original problem of logical omniscience for remote truths.

[7] For a very different sort of objection to a doxastic probabilistic view (a view suggested by Easwaran, but very similar to Williams's view), see Elga & Rayo, "Fragmented Decision Theory", fn. 22, (version of Sept 2, 2016).

[8] In an article that is something of a companion paper to this one, I develop this positive view at a bit more length. I also do a bit to discuss this view in the context of some empirical psychology. See Dogramaci 2018.

hesitant to fully embrace it, and then I'll propose and try to support a new view, a more general "dynamic" framework (I'll explain what I mean), one that preserves and incorporates Hacking's insights.

I'm also going to combine Hacking's idea (to tie credal norms to knowledge) with another important idea, the one we already heard from by Gilbert Harman: you only acquire rational requirements if you are, or ought to be, *interested in* the propositions in question. Or, other formulations I take to say the same thing: … only if you do, or ought to, *consider* the propositions, or turn your *attention* to them. (Harman uses "considering" when he first brings up the point (15) but then switches to talking of when you "have or ought to have an interest" in the propositions. Steinberger, forthcoming, who defends and applies Harman's idea, sticks with "considers". John Broome puts Harman's idea in the equally reasonable terms of "caring", pp. 157-8.) Harman's "considering" / "interest" restrictions make a very important positive difference to the plausibility of the theory that results.

I'll also amend Hacking's proposal in the following way: I'll say the correct norms depend not on what the agent knows, but on what they are *in a position to* know, or for brevity I'll talk of what we "can" know. This notion of what you are in a position to, or can, know is widely used in epistemology, but there's no obvious analysis, so I'll rely on familiarity with it through its theoretical utility elsewhere. (See e.g. Williamson 2000, esp. p.95.) An important assumption I make about the notion is that we're *not* in a position to know remote consequences, like Fermat's theorem, without a proof, though we can know things that follow by simple deductive steps (e.g. Kelly can know she's in danger). We need to use this notion, "can know", in our proposed norms rather than just "know", otherwise by simply refusing to believe (and thus know) something you could get off the hook too easily.

So amended, and otherwise paraphrased just slightly for clarity, the Hacking proposal (or perhaps better the "Hacking-Harman" proposal) for the norms is as follows, written here as if they replace (A1) and (A3) — (A2), Non-negativity, is left alone:

> (H1): if you have considered *p*, and you can know *p*, then the rational credence in it is 1.
> (H3): if you have considered *p* and considered *q*, and you can know that they are not both true, i.e. *~(p&q)*, then the rational credence to give their disjunction is the sum of your credences in the disjuncts.

Hacking just gave those two. I think there are many more such static norms, just as basic as the above two. (I call a norm "basic" to mean it is not derivable from other norms.) We can, for example, propose plausible static norms that correspond to the other laws of probabilism and which incorporate Hacking's knowledge constraints:

> (H-Entailment): if you have considered *p* and considered *q*, and you can know that it's not the case that *p&~q*, then the rational credence to give *q* is at least the rational credence you give to *p*.
> (H-Partition): if you have considered a set of propositions, and you can know that exactly one of them is true, then the rational credences to have in those propositions add up to 1.

Below I'm going to argue that there are more basic norms of a different sort, dynamic norms, in the full explanatory theory of rationality, but at this point we can already see that the above "static" norms accommodate the motivation Williams had for his own view. When a paradox leaves you in rational doubt about whether some instance of P v ~P is a logical truth, you will also be in rational doubt about whether P v ~P is a truth at all, and thus you will not be in a position to know it, and so, there is no violation of rationality.

Why, when I wrote out these H norms, didn't I mention necessity or entailment? Why did I say "you can know that it's not the case that *p&~q*" instead of something closer to the traditional idea, like "you can know *q* is a consequence of *p*"? Similarly, why not talk explicitly of exclusion and partitions? I did, after all, keep those suggestive terms in the names of the two rules "H-Entailment" and "H-Partition". Why drop all talk of logic and modality? (We could say these clauses I used are a species of "material" modality, but I'm not sure that's a helpful observation.) The reason I didn't put the norms in modal terms is that I don't think the most plausible basic norms here require knowledge of any modality, not logical, metaphysical or epistemic modality. But it is true that *if* you knew any kind of such necessity, that would trigger the above H norms, since knowledge of such necessities can rationally commit you to the various conditions I used in my clauses. For example, if you know that certain propositions are, say, metaphysically or nomologically exclusive, or form a partition of the metaphysical or nomological possibilities, then H-Partition will kick in for you. Even if the modality is merely epistemic—the kind Williams used—the norms kick in. For example, if you can know that *p&q* must be false, in the same epistemic sense of "must" that Williams was discussing, this triggers (H3). The critical difference with Williams is that you need to *know* this thing that we use an epistemic modal to express. (There's a million interesting and difficult philosophical questions about knowledge of epistemic modals, but we don't need to settle them before we endorse the present view. All endorsement are defeasible anyway!)[9]

Why haven't Bayesians followed Hacking's proposal, or anything closely inspired by it like our above amended version? I can only speculate on this partly sociological question, but my guess is that they worry it would disrupt the traditional Bayesian solutions to some of the paradoxes of confirmation theory.[10] For example, Bayesians suggest their view can solve the paradox of the ravens, where the solution involves showing that an observation of a white shoe does confirm that all ravens are black, but only very, very little. But to show such facts about confirmation, Bayesians run through calculations using traditional probabilism, ignoring constraints like the knowledge constraints Hacking imposes. One take on the situation here is that Bayesians believe that neither their solution, nor even a similar solution, could be provided

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[9] I said Hacking's view is critically different from Williams's, but Brian Knab suggested to me that it may be a more useful perspective to think of Hacking's approach as simply a way of adding interpretive or normative constraints on Williams's view, constraints that make the doxastic worlds become worlds that are "epistemically possible" in the sense of being left-open-by-your-knowledge.

[10] This appears to be Talbott's speculation in his SEP entry. "Hacking and Garber have made proposals to relax the assumption of logical omniscience. Because relaxing that assumption would block the derivation of almost all the important results in Bayesian epistemology, most Bayesians maintain the assumption of logical omniscience and treat it as an ideal to which human beings can only more or less approximate." (Sec. 6.1.A)

if we use Hacking's view. Another take is that Bayesians would be happy to run some similar solution that Hacking's view allowed for, but they felt it would be easier and acceptable to just work with the more straightforward calculations that traditional probabilism allows. (The latter hypothesis is what explains why Hacking didn't even mention his own theory when he later wrote a textbook on probability theory. The last paragraph of his 1967 paper advises future textbook writers—himself included, it turned out—not to fuss over revising the standard textbook presentation of probability theory.) My hope is that your favorite traditional solution to your favorite paradoxes can still be run, in some similar way, on Hacking's view. But of course it will depend on the details in each case.[11]

Hacking's proposed norms are *static* norms. They are (as amended) plausible, but, as I said, I think they give us an incomplete picture. They're only one part of the theory of rational credence. We need *dynamic* norms, as well as static norms. Let me now clarify this distinction. When I talk about "dynamic" norms, I'm talking about principles that say how a rational person —a person with rational attitudes—may, or must, revise their attitudes by engaging in *reasoning*. And when I talk about reasoning, I'm talking about *basing*; I'm talking about the adoption, or rejection, or re-establishing, of an attitude on the *basis* of other attitudes (or perhaps other mental states, e.g. maybe experiences). Probabilism makes no mention of reasoning or basing. So, we can call probabilism or any such norm a "static" norm. It tells you how your attitudes, at any given time, should have ended up, but doesn't comment on how you reasoned your way there.

The proposal I want to advance here is motivated by this general idea: any true static norms will be entailed and explained by the true dynamic norms. I won't give an argument for this general idea, but I will rely on it as a reasonable motivation for the proposal I want to develop, which conforms to the general idea. (John Broome's book, *Rationality Through Reasoning*, makes a strong case for the general idea.)

To see that we need dynamic norms, and to see what they may look like, let's briefly turn away from credence and look at the rationality of full beliefs for a moment.

Full beliefs are also subject to static norms. For example, here is a static norm concerning consequence:

> (F-S-Entailment): if you are considering the propositions p and q, and you can know that q is a consequence of p, then it's irrational to believe p and disbelieve q.

But the theory of rational full belief must also include something that explains how a rational person reasons. There are, in other words, dynamic norms. Here is a plausible suggestion for a dynamic norm (underlining is to help readability):

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[11] A rare discussion of Hacking's view is Elga & Rayo, "Fragmented Decision Theory", (ms). They raise a good objection to Hacking's view, and propose to replace it with a "fragmented" decision theory, which is a theory that attributes different credence functions to agents depending on what choice is before them in a given context. I actually find Elga & Rayo's objection and proposal sensible, but incorporating it into the present paper would make things complicated. I'll trust that others will be able to see how the view I'll propose and the view Elga & Rayo propose can be taken as fragments that should be integrated in the correct overall view.

> (F-D-Entailment): if you're considering the propositions *p* and *q*, you rationally believe *p*, you can know that *q* is a consequence of *p*, and you lack any reasons to disbelieve *q*, then you're rationally required to infer *q* (or, if you already believed it, to re-establish *q*).

There are also some related dynamic norms including one requiring you to infer ~*p* when you *do* have sufficient reason to believe ~*q*. I won't write it all out. These dynamic norms for reasoning are what the logical proof rules of Modus Ponens and Tollens mirror, but since we are discussing *reasoning*, not logic, we need these clauses about your lacking defeating reasons and the bases being known or rationally believed. (I don't know how to give principles that fully explain when you have these defeating reasons I'm mentioning. But the phenomenon is intuitive.)

The static norm (F-S-Entailment) is entailed and explained by the dynamic norms (F-D-Entailment) and its "tollens" counterpart.[12] It's plausible that the situation is analogous for credences. This analogy is my main reason for believing that there are dynamic norms for credences, and that they entail and explain the static norms.

What might the dynamic norms for credence be? Here are some suggestions that I hope will look plausible. Even if they're not exactly correct, I hope they at least illustrate the general project here in a way that supports thinking it's on the right track.

> (HD-1): if you are considering *p* as well as evidence in virtue of which you can know *p*, then you are rationally required to adopt (or re-establish) a credence of 1 in *p* on the basis of that evidence.
> (HD-3): if you are considering *p* and considering *q*, and you can know that *p*&*q* is false, then you're rationally required to, on the basis of that knowledge, revise (or re-establish) your credences in the disjunction and the disjuncts so that your credence in the disjunction is the sum of your credences in the disjuncts.
> (HD-Entailment): if you're considering *p* and considering *q*, and you can know ~(*p*&~*q*), and your credences in *q* is lower than your credence in *p*, then you're rationally required to, on the basis of that knowledge, raise your credence in *q* or lower your credence in *p* so that the former is no less than the latter.
> (HD-Partition): if you're considering a set of propositions and you can know exactly one of them is true, you're rationally required to, on the basis of that knowledge, revise your credences in those propositions so that your credences add up to 1.

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[12] Declan Smithies, in helpful comments, pushed me on why it cannot be that the static norms are fundamental and explain the dynamic norms. On this suggestion, the dynamic norms are true *because* there must be some way of reasoning that has you fulfill the static norms, and the dynamic norms just tell you to reason in some such way. I reject this view because I think the complete specification of all the dynamic norms will require you to reason in fairly specific ways to fulfill the static norms; the mere fact that you must reason in some way to fulfill a static norm doesn't entail what specific ways you must reason. For example, (F-S-Entailment) could be fulfilled by reasoning in some weird and complex way that doesn't look rational at all but still has you end up obeying that static norm. But you have to obey (F-S-Entailment) by reasoning in certain ways, like the Modus Ponens/Tollens way that (F-D-Entailment) indicates. So, the more specific dynamic norms entail the more general static norms, but not vice versa.

(It would be nice to also have principles that tell us some "only if", i.e. necessary, conditions on when to adopt certain credences, but these are a bit hard to find. For example, you can have credence 1 in *p* even when you don't know it. One case is when ~*p* is one of infinitely many equally likely outcomes, for example when God is picking a random natural number and *p* is the proposition that they'll pick 42. Another problem: I think I should have credence 1 that I have hands because I'm in a position to know that (and Clarke and Greco CITE convinced me credence 1 is not problematic), but then, as Declan Smithies reminded me, I should also recommend credence 1 for the poor handless brain in a vat, even though they're not in a position to know they have hands. One option is to propose principles concerning not what we're in a position to know but what we're in a position to justifiably believe, but I'll not speculate further here about the right necessary conditions on having these rational credences.)

It may seem that, by gesturing at so many candidates for dynamic norms on credal reasoning, I'm only making it less convincing that these are *basic* norms (i.e. are underived from any other norms) that constitute part of the correct theory, a theory that replaces probabilism with its very short and elegant list of axioms, A1 - A3. But that would be a mistake; it would be a mistake to expect the correct theory of rational reasoning to feature such a short and elegant list of basic principles. This mistake naturally arises if you confuse being a good candidate for a mathematical axiom and being a good candidate for a basic epistemic norm. What makes a principle a suitable candidate for being an axiom is something to do with its mathematical interest or practical use in mathematics, and certain such interests or practical uses pressure mathematicians to seek out the shortest and most elegant sets of axioms that generate the desired theory. But the basic norms governing rational reasoning are often inelegant, for example they may include logical or mathematical redundancies. HD-Partition is (in a sense) logically redundant given the other norms, and the others are likewise redundant given HD-Partition, but I take this as no reason to think HD-Partition is any less fundamental, any less likely to feature as a basic norm. Natural examples of rational reasoning with credences seem to be nicely explained by it, with the explanation *not* running through any other more basic norms. For example, suppose the detective is wondering what the chance is that the butler committed the murder and the only other possible suspect is the maid. If the detective knows that just one of them did it, i.e. these are two "exclusive and exhaustive" possibilities, then the detective's credences in the two should add up to one. (What if it's possible that the butler and the maid were in on it together, or possible that the death wasn't a murder at all? What if you or I know these things are possible? In that case, we'd have to agree, the detective did not *know* that the two possibilities of the butler's being the murderer and maid's being the murderer form a partition, that exactly one of these possibilities is true.)

We should and can also add norms to govern conditional credence. People have conditional credences, and there are static norms governing their interrelations with each other and with unconditional credences. But there must be dynamic norms here too. But first, what does a plausible static norm for conditional credence look like?

A famous traditional static norm for conditional credence, one that is often included as a component of traditional probabilism, is that rational credences should obey Bayes' theorem.

(Bayes' theorem): Pr(p/q) = Pr(q/p) * Pr(p) / Pr(q)

But as with the rest of probabilism, I think the more plausible norms here should include some modifications. I don't think a rational person, even an "ideally" rational person, must *always* obey Bayes' theorem. Continuing with our main guiding idea: you need to follow dynamic norms of reasoning before you obey Bayes' theorem. But before we get to that, what is the more moderate, plausible *static* norm here?

Hacking's constraints don't appear to apply; those concern the person's knowledge of some (loosely put) "modal" facts, and Bayes' theorem doesn't appear to involve any suitable candidates. But what will help here are Harman's interest/considering constraints.

> (H-Conjunction): if you're considering *p*, *q*, and their conjunction, and you're rational, then your credence in the conjunction should be the product of your conditional credence in one-conjunct-given-the-other times your unconditional credence in the given conjunct.

Now we need to spend a bit of time dealing with a worry about this. (H-Conjunction) may still look too demanding, not much of an improvement over a requirement to obey Bayes's theorem itself. How could this claim about multiplying two credences together be a basic, underived norm that rational people need to meet?

I think (H-Conjunction) is right, but to bring out its plausibility we need to re-state it.

First, let's give it an equivalent statement that replaces talk of credences with talk of beliefs about chances. The equivalence of such attributions is defended in other work, recently especially in papers by Yalcin (CITES). (What are these "chances"? They are epistemic probabilities; see the cited work for elaboration and defense.)

> (H-Conjunction): if you're considering *p*, *q*, and their conjunction, then, if you're rational, you believe the chance of *p&q* is the same as the conditional chance of *p-given-q* times the chance of *q*.

But you might still worry. A belief that one chance is the product of two other chances may still look too sophisticated to plausibly be required of rational people. Must rational people have beliefs about the products of chances? I think this norm that says they must can be made yet more palatable by a re-statement one more time in again equivalent language. Chances (epistemic probabilities) just are certain numbers. In particular, they're all proper fractions. They're fractions of the (certain) chance of the total set of possible outcomes. We can intuitively talk of these fractions as fractions of that total set. So, we can re-state the norm using the language of fractions as follows.

> (H-Conjunction): if you're considering *p*, *q*, and their conjunction, then, if you're rational, you believe the fraction of possible outcomes where *p&q* is true is the same as the fraction of outcomes where *p-supposing-q* is true among the fraction of the outcomes where *q* is true.

At this point, even if, stated as abstractly as it is, the norm looks very demanding, I think looking at a concrete example can finally show that it's intuitive enough to be a plausible basic norm.

Suppose Sal is wondering what the chance is that his new neighbor is a gay republican. Sal hasn't met the guy yet, so all he has to go on are his beliefs about the general demographic statistics. Sal's wondering about the chance of a conjunction: my new neighbor is gay and they're a republican. I think it's intuitive that Sal needs to end up the way our norm requires him to end up. He needs to consider the chance his neighbor is-gay-supposing-he's-a-republican *times* the chance his neighbor is a republican. Even more comprehensibly: one thing he needs to consider (in the total space of possibilities here, e.g. maybe of people who are possibly his new neighbor) is the fraction *of republicans* who are gay, but that is not all; he needs to furthermore consider that as a fraction of the fraction of people (among all people) who are republicans. In effect, (H-Conjunction) codifies the intuition that the fraction of possibilities where *p&q* is true is a certain "fraction-of-a-fraction", namely the fraction of *p-supposing-q* cases among the fraction of *q* cases. That last "among" is where the calculation of a product occurs when a rational person obeys the (H-Conjunction) norm. If it's, as I hope, intuitive that rational people need to manage their credences in the way described here, then the norm is a good candidate as a basic static norm for rational credence.

If (H-Conjunction) isn't a basic norm, then either it's off-base or it can be derived from something more basic. But neither of those options looks very likely. So, I conclude it's a basic static norm.

In describing the example about Sal and his neighbor, we've already indicated the outlines of the dynamic norm that entails and explains the above static norm.

> (HD-Conjunction): if you're considering *p*, *q*, and their conjunction, and you're rational, then revise or re-establish your credences so that your credence in the conjunction equals the product of your conditional credence in one-conjunct-given-the-other times your unconditional credence in the given conjunct.

Just because it's very demanding to *uncover and make explicit* the abstract and somewhat complex content of the norm, that doesn't mean the norm is too demanding to plausibly *govern* the thinking of rational people. The theorist's task in stating the norm may be demanding without making the norm itself too demanding on rational thinkers.

The dynamic norm is pretty similar to the static norm in this case; the important addition is just the requirement to *revise* (or re-establish) credences, when you're considering the probabilities of conjunctions and related conditional probabilities, so that they match up the right way. But it's important to an adequate theory to include the dynamic norm. The dynamic norm explains that we need to end up the way the static norm requires *by reasoning*. Suppose Sal initially thinks, "About a tenth of people are gay, and about half are republicans, so the chances are about a twentieth that my new neighbor is a gay republican." And then Sal's friend points out that Sal is making a mistake, since he didn't entertain the *conditional* chance of someone's being gay *supposing* they're a republican, which is the relevant chance to consider and is, I presume, much smaller than one tenth. (H-Conjunction) correctly claims that Sal's attitudes are irrational, but that's true *because of* what (HD-Conjunction) says, namely that Sal needs to either lower his credence that his neighbor is a gay republican, or increase his conditional credence in a person's being gay supposing they are a republican (or both, or revise his one half credence the neighbor is a republican.)

The wide-scope instruction issued by (HD-Conjunction) leaves open several options for how Sal should revise. This is analogous to the situation with full beliefs. A static norm prohibits believing *p*, ~*q*, and if-*p*-then-*q*, but it's a difficult and unsettled question when a rational person ought to reason by "ponensing" or "tollensing", a question that no epistemologist has given an adequate theory of. So I'm going to leave things similarly unsettled for how we ought to rationally revise our credences.

There's actually another dimension of symmetry concerning how Sal can rationally reason to satisfy (H-Conjunction): he can, of course, swap the *p*'s and *q*'s around. That is, he can ask himself the chance his neighbor is a-republican-supposing-they're-gay, and take the product of that with the chance they're gay. This reflects a basic norm of rational mathematical thinking: a "fraction-of-a-fraction" is a commutative property, in the sense that it's indifferent to which fraction you take first. This is how Bayes' theorem actually governs rational thinking. By obeying the (H-Conjunction) norm in both of these two equivalent ways, a rational person ends up connecting the two conditional credences that are converses of one another. You don't *have to* consider all these conditional and unconditional credences, but if you do, (H-Conjunction) will require you to manage them correctly. That's the rational core of Bayes' theorem as it amounts to a static norm on rational thinking.

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Bibliography

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